Money Math for Teens

Revenue, Expenses and the Break-Even Point
Revenue, Expenses and the Break-Even Point

Lesson Plan

OBJECTIVE
Introduce students to fundamental concepts of starting and running a business successfully. Focus on revenue, expenses, expansion and use of break-even point analysis.

Students will be able to:

- Understand the vocabulary associated with revenue and expenses
- Understand the relationship between revenue, expenses and profit
- Analyze break-even data in a variety of situations
- Calculate price, sales units and revenue values for different situations
- Design a plan for success
- Plan for expansion.

TEACHING MATERIALS

- Lesson plan with precalculated analytical examples
- Revenue, Expenses and the Break-Even Point student handout
- Student assessment worksheet with solutions
- Teaching tool: A public service announcement is available to support this lesson at [www.saveandinvest.org/financialbasics/teens](http://www.saveandinvest.org/financialbasics/teens) (scroll down to the video “Understanding the Break-Even Point”).

LESSON ACTIVITY

1. Determine students’ prior knowledge of fundamental vocabulary and concepts by asking questions such as:
   - If you wanted something you can’t afford, how would you get it?
   - What are start-up costs? Provide examples.
   - What is profit, and where does it come from?
     - Revenue above and beyond expenses.
   - What is revenue, and how does it differ from income?
     - Revenue is earned income from a business or occupation. Income can be passive, such as interest and dividends.
   - What is your understanding of break-even point?
2. Present the student handout.

Note: Depending on grade level, content may require further explanation.

- Introduction of the fundamental premise (page 7 of handout):
  - Trevor has a goal, and he’s motivated to accomplish it.
  - Trevor is a good organizer.
- Ask for student feedback:
  - Has Trevor forgotten anything?
  - Are there some things you would do differently?
- Review basic business terminology and formulas with students (page 8 of handout):

\[
\text{Revenue} = \text{Expenses} + \text{Profit}
\]

or

\[
\text{Revenue} = \text{Price charged for a service} \times \text{Number of clients}
\]

Revenue = \( P \times x \), where:
- \( P \) = Price
- \( x \) = Number of clients

\[
\text{Expenses} = \text{Fixed costs} + \text{Variable costs}
\]

Expenses = \( FC + Vx \), where:
- \( FC \) = Fixed costs
- \( V \) = Cost per client
- \( x \) = Number of clients

BEP (Break-even point): Revenue = Expenses with no profit

Profit = Revenue in excess of expenses

- Break down expenses to fixed and variable expenses.
  - **Fixed expenses** are costs that do not vary based on the number of units sold, such as Trevor’s agreement for use of lawn equipment.
  - **Variable expenses** are related to production of each unit, such as gasoline to power equipment.
- Trevor needs to determine a price for his services. He starts by figuring out how many lawns he must cut to break even (see the table starting on page 9 of the handout). This number represents his break-even sales units.
Revenue, Expenses and the Break-Even Point

- **Break-even sales units** are the number of units that must be sold to reach the break-even point. It can be calculated as follows:

\[ P_x = V_x + FC \]

\( X \) is the number of units sold that brings the equation into balance.

Use algebra to solve for \( X \):

\[ P_x = V_x + FC \]

\[ (P_x - V_x) = FC \]

\[ x(P - V) = FC \]

\[ X = \frac{FC}{(P - V)} \]

Use the formula for calculating break-even sales units to verify that the data in the table is correct. For example:

<table>
<thead>
<tr>
<th>Lawns Mowed</th>
<th>Expenses (FC + Vx)</th>
<th>Price Charged to Break Even</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>50 + (1.90)(5) = $59.50</td>
<td>$11.90</td>
</tr>
</tbody>
</table>

\( X = \) Lawns mowed = 5

\( FC = \) Fixed costs = 50

\( V = \) Variable costs = Cost of ½ gallon of gasoline per lawn = 1.90

\( P = \) Price charged = 11.90

\[ X = \frac{FC}{(P - V)} = \frac{50}{(11.90 - 1.90)} = \frac{50}{10} = 5 \text{ lawns} \]

- The **contribution margin** is the amount of revenue available for use in paying the business’s fixed costs (see page 9 of the handout).

**Contribution margin = Revenue – Variable costs**

- Subtracting the per-unit costs (variable costs) from the revenue earned leaves the contribution margin.

- Once Trevor’s contribution margin exceeds $50, he will be making a profit that can be put toward his bike.

- The table on page 10 of the handout shows contribution margins for four different prices Trevor could charge and four different customer points. Review the table with students.
Revenue, Expenses and the Break-Even Point

• Comprehension check (page 10 of the handout):
  • What would Trevor’s contribution margin be if he charged $25 per lawn and was cutting eight lawns per week?
    Revenue = Price \times Clients = 25 \times 8 = 200
    V = 1.90(8) = 15.20
    \[ CM = Revenue - Variable\ cost = 200 - 15.20 = \$184.80 \]
  • What if he charged $20 per lawn and had a dozen clients each week?
    Revenue = Price \times Clients = 20 \times 12 = 240
    V = 1.90(12) = 22.80
    \[ CM = Revenue - Variable\ cost = 240 - 22.80 = \$217.20 \]

• Availability: How many lawns can Trevor cut in one week (pages 10–11 of the handout)?
  Four hours of light (after school) = 240 minutes
  50 minutes per lawn (40 to cut, 10 minutes to reposition to next lawn)
  Four lawns = 200 minutes with 40 minutes left to cut one additional lawn
  Trevor will finish when it’s dark.
  • Trevor can service five lawns per afternoon, or 25 lawns in a five-day work week.
  • Trevor can’t service all 200 clients if everyone who received a flier hired him.

• If Trevor can’t successfully take on all 200 clients, how many can he actually handle?
  • 25 clients per week.

• Trevor fills his schedule, charging $20 per lawn. What will be his contribution margin? Is he making a profit?
  \[ X = Clients = 25 \]
  \[ P = Price = 20 \]
  \[ Px = Revenue = 20 \times 25 = 500 \]
  \[ Vx = 1.90 \times 25 = 47.50 \]
  \[ CM = Revenue - Variable\ cost = 500 - 47.50 = \$452.50/week \]

Revenue = Expenses + Profit
500 = 50 + 47.50 + Profit
500 = 97.50 + Profit
Profit = 500 – 97.50 = $402.50/week
Yes, Trevor is profiting.
• If we assume his bike (including tax and accessories) costs $500, how long will it take for Trevor to be able to purchase it?
• A profit of $402.50 per week will allow Trevor to purchase the bike before his second full week of servicing clients is complete.
• After one week, Trevor will have made $402.50 profit.

**Week Two, Day One**

Revenue = Expenses + Profit
20 x 5 = 50 + (1.90 x 5) + Profit
100 = 50 + 9.50 + Profit
100 – 59.50 = Profit
Profit = **$40.50**

402.50 (first week’s profit) + 40.50 (week two, day one profit) = **$443 profit so far**

**Week Two, Day Two**

100 = 1.90 x 5 + Profit (fixed cost for the week is paid)
Profit = 100 – 9.50
Profit = 90.50

443 (profit so far) + 90.50 (week two, day two profit) = **$533.50**

By the end of day two of the second week, Trevor will have made **$533.50** toward the bike.

• Assuming a full schedule, charging $25 per lawn, recalculate his contribution margin, profit and time until bike purchase.

\[
X = \text{Clients} = 25 \\
P = \text{Price} = 25 \\
Px = \text{Revenue} = 25 \times 25 = 625 \\
Vx = 1.90 \times 25 = 47.50 \\
CM = \text{Revenue} - \text{Variable cost} = 625 - 47.50 = \$577.50/\text{week} \\
\]

Revenue = Expenses + Profit
625 = 50 + 47.50 + Profit
625 = 97.50 + Profit
Profit = 625 – 97.50 = **$527.50/week**

A profit of **$527.50 per week** will allow Trevor to purchase the bike after one week of work.

• What would be the pros and cons of each price decision? Expected discussion responses:
  • $25 would generate more revenue and more profit.
  • $25 would allow Trevor to purchase the bike two days quicker.
  • $20 would make his service more attractive and would help him fill his schedule more quickly.
3. **Lead a class discussion on Trevor’s options (page 11 of the handout).**
   Suggested questions follow, but student perspectives in other directions may lead to interesting conversation.
   Let’s say Trevor fills his schedule and keeps happy clients.
   - Do you think he will continue to grow his client base?
   - Instead of Trevor refusing work, what else can he do?
   - Trevor could one day expand outside his neighborhood.
     - What would he have to consider before expanding his geographic area?
     - What numbers would change if he expanded?

4. **Evaluate students’ comprehension (see assessment worksheet).**
Trevor is a typical high school student. He gets good grades, takes school seriously and understands that hard work will make for a successful future. He would like to purchase a new high-tech bike. However, it will take several hundred dollars and a lot of hard work for him to get it. One of Trevor’s chores around the house is to maintain the lawn. He has become very good at it and decides to profit from his skill by starting a lawn mowing business. He starts by planning his business strategy and outlines some facts.

- He’ll need equipment for the business: a lawn mower, weed wacker and broom or leaf blower.
- The cost to rent this equipment, even used, will be several hundred dollars.
- He will need to sign an agreement to rent the equipment.
- He’ll need gasoline to power the equipment.
  - The average price of gasoline is $3.80 per gallon.
- Clients must be located near his home.
- He has no transportation to move himself and his equipment to each client.
- Clients must be close enough for him to easily transport his equipment and get the job done in a reasonable amount of time.
- He’ll focus his marketing efforts in his neighborhood of approximately 200 homes.
- He has school commitments and time restrictions.
  - He can’t let the business affect his grades.
  - School ends mid-afternoon.
    - Cutting lawns requires daylight.
    - He will have about four hours to work after school.
    - It takes him about 40 minutes to cut his family’s lawn, and it’s similar in size to others in the neighborhood.
  - Weekends will be saved for makeup work in case a day during the week is rained out.

Trevor creates a to-do list:
1. Secure an agreement to rent equipment.
2. Create a flyer announcing the creation of his new lawn mowing business.
3. Distribute flyers throughout the neighborhood.
4. Prepare to work immediately.
Has Trevor forgotten anything? Are there some things you’d do differently?

Over the next few days, Trevor makes progress. He secures an agreement to use lawn equipment consisting of a lawn mower, a weed wacker, a leaf blower, two one-gallon gas cans (one for gas to power the lawn mower and one for a gas/oil mixture to power the weed wacker and leaf blower) and a five-gallon gas can for refueling. He has agreed to pay $50 per week to use this equipment.

Trevor designed a flyer announcing his new business and has distributed flyers to the 200 homes in his neighborhood.

Now Trevor needs answers to some questions:

- How much money will I make each week?
- How many lawns must I cut to cover my costs?
- How much profit will I make?
- How long will it take me to earn enough for my bike?

Trevor recalls some basic business terminology and formulas he learned in school:

\[
\text{Revenue} = \text{Expenses} + \text{Profit}
\]

\[
\text{Revenue} = \text{Price charged for a service} \times \text{Number of clients}
\]

\[
\text{Revenue} = P \times X, \text{ where:}
\]

\[
P = \text{Price}
\]

\[
X = \text{Number of clients}
\]

**Expenses = Fixed costs + Variable costs**

\[
\text{Expenses} = FC + Vx, \text{ where:}
\]

\[
FC = \text{Fixed costs}
\]

\[
V = \text{Cost per client}
\]

\[
X = \text{Number of clients}
\]

**BEP (Break-even point)**

\[
\text{Revenue} = \text{Expenses with no profit}
\]

\[
\text{BEP} = \text{Revenue} = \text{Expenses}
\]

\[
Px = FC + Vx
\]
Revenue, Expenses and the Break-Even Point

His **fixed costs** are $50 per week (the cost he agreed to pay to rent the equipment). This cost will not change regardless of how many lawns he mows.

His **variable cost** will be the cost of the gasoline he uses to cut each lawn. Typically, Trevor uses ½ gallon of gas to fill the mower and the other equipment to cut his family’s lawn. Gas costs $3.80 per gallon, so Trevor’s variable cost will be $1.90 per lawn.

Trevor’s expenses can be represented as $\text{FC} + Vx = 50 + (1.90)x$, where $X$ equals the number of lawns he cuts.

How much should Trevor charge to cut one lawn? What will be his BEP?

<table>
<thead>
<tr>
<th>Lawns Mowed</th>
<th>Expenses $\text{FC} + Vx$</th>
<th>BEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 + (1.90)(1) = $51.90</td>
<td>$51.90</td>
</tr>
<tr>
<td>2</td>
<td>50 + (1.90)(2) = $53.80</td>
<td>$26.90</td>
</tr>
<tr>
<td>3</td>
<td>50 + (1.90)(3) = $55.70</td>
<td>$18.57</td>
</tr>
<tr>
<td>4</td>
<td>50 + (1.90)(4) = $57.60</td>
<td>$14.40</td>
</tr>
<tr>
<td>5</td>
<td>50 + (1.90)(5) = $59.50</td>
<td>$11.90</td>
</tr>
</tbody>
</table>

Trevor sees that having just one client would mean he would have to charge more than $50 to cut the lawn to break even. Pricing his service above $50 would make his service expensive and would limit his ability to attract more customers. The lower his price, the more customers he may get. But Trevor is starting the business to make a profit, not to break even, so how can he balance his price and still make a profit? Trevor remembers an important term used in BEP analysis: **contribution margin** (the amount of revenue available for use in paying the business’s fixed costs).

**Contribution margin = Revenue – Variable costs**

Subtracting the per-unit costs (variable costs) from the earned revenue leaves the contribution margin.

- If the contribution margin is less than the fixed costs, the break-even point has not been reached, and money is lost.
- When the contribution margin equals fixed costs, the BEP is reached.
- If the contribution margin exceeds fixed costs, there is a profit.

Once Trevor’s contribution margin exceeds $50, he will be making a profit, which he can put toward the purchase of the bike.

Trevor needs to calculate the contribution margin at different price levels to determine the ideal price to charge for cutting each lawn.
Revenue, Expenses and the Break-Even Point

Contribution Margin for Price Charged

<table>
<thead>
<tr>
<th>Price Mowed</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30</td>
<td>60 – 3.80 = $56.20</td>
<td>90 – 5.70 = $84.30</td>
<td>120 – 7.60 = $112.40</td>
<td>150 – 9.50 = $140.50</td>
</tr>
<tr>
<td>$25</td>
<td>50 – 3.80 = $46.20</td>
<td>75 – 5.70 = $69.30</td>
<td>100 – 7.60 = $92.40</td>
<td>125 – 9.50 = $115.50</td>
</tr>
<tr>
<td>$20</td>
<td>40 – 3.80 = $36.20</td>
<td>60 – 5.70 = $54.30</td>
<td>80 – 7.60 = $72.40</td>
<td>100 – 9.50 = $90.50</td>
</tr>
<tr>
<td>$15</td>
<td>30 – 3.80 = $26.20</td>
<td>45 – 5.70 = $39.30</td>
<td>60 – 7.60 = $52.40</td>
<td>75 – 9.50 = $65.50</td>
</tr>
</tbody>
</table>

The more clients Trevor has, the less he can charge for his service and still make a profit. His contribution margin must exceed $50 for Trevor to make a profit. From the table, it is clear that if he only has two lawns to mow, he must charge at least $30 per lawn to make a profit. He could charge $20 per lawn with three clients and $15 per lawn once he has more than three clients.

Comprehension Check

- What would Trevor’s contribution margin be if he charged $25 per lawn and was cutting eight lawns each week?
- What if he charged $20 per lawn and had a dozen clients each week?

In order for Trevor to determine the best price for his service, he must first know how many clients he will have. What if each of the 200 homeowners in his neighborhood hired him? If it takes him 40 minutes to cut a lawn properly and another 10 minutes to transport his equipment to the next client, could Trevor service all 200 clients?

Another fundamental business practice is quality customer service. Trevor realizes that not every day will be suitable for mowing lawns; it is inevitable that weather will eventually cancel his daily commitments. As backup, he keeps Saturdays open. That will leave him eight hours available to catch up should weather postpone his appointments during the week. If it rains on Monday, he’ll cut his Monday lawns on Tuesday, his Tuesday lawns on Wednesday, and so on. Saturday could be used to cut his Friday lawns and still service all of his weekly clients.
Revenue, Expenses and the Break-Even Point

- If Trevor can’t successfully take on all 200 clients, how many can he handle?
- Trevor fills his schedule, pricing his service at $20 per lawn. What will be his contribution margin?
  - Is he making a profit?
  - If we assume his bike (including tax and accessories) costs $500, how long will it take for Trevor to be able to purchase it?
- Assuming a full schedule, charging $25 per lawn, recalculate his contribution margin, profit and time until bike purchase.
- What would be the pros and cons of each price decision?

Let’s say Trevor fills his schedule and keeps happy clients.
- Do you think he will continue to grow his client base?
- Instead of Trevor refusing work, what else can he do?
- Trevor could one day expand outside his neighborhood.
  - What would he have to consider before expanding his geographic area?
  - What numbers would change if he expanded?
Assessment: Revenue, Expenses and the Break-Even Point

1. Define revenue.
   A. The price charged for a service
   B. Income in excess of expenses
   C. Total income produced by selling a product or service
   D. The income necessary to pay all expenses

2. What is the break-even point?
   A. The point where revenue exactly equals expenses; there is zero profit
   B. Expenses that are not affected by how many products are sold
   C. Expenses of producing or delivering each product or service
   D. The point where zero profit has been earned

3. The expenses of running a business are:
   A. Fixed and recurring
   B. Variable and constantly changing
   C. Unpredictable
   D. A combination of fixed and variable per-unit

4. Contribution margin is:
   A. Revenue available to only pay fixed costs
   B. Equal to fixed costs at the break-even point
   C. Calculated by subtracting variable costs from total revenue
   D. All of the above

5. With equipment and facility rental costs of $525/month and employee wages of $120/week per employee, Trevor’s Lawn Service has a contribution margin of $2,640 on monthly revenue of $3,175. What are Trevor’s Lawn Service’s monthly variable costs?
   A. $1,005
   B. $535
   C. Cannot be determined without knowing how many employees
   D. Cannot be determined without knowing the number of monthly sales (lawns mowed)
6. If Trevor's Lawn Service employs four full-time workers, and they service 107 lawns each month, what are the per-lawn variable costs?
   A. $5  
   B. $535  
   C. $1,920  
   D. $2,445

7. If Trevor's Lawn Service charges $38.95 to cut a lawn, how many lawns represent the company's break-even point?
   A. 63  
   B. 19  
   C. 72  
   D. 30

8. How much revenue would it take for Trevor's Lawn Service to break even if they cut 107 lawns?
   A. $3,632.65  
   B. $2,804.40  
   C. $2,444.40  
   D. $2,980.00

9. What would Trevor's Lawn Service's profit be if they charged $38.95/lawn and cut 107 lawns?
   A. $1,723.25  
   B. $1,363.25  
   C. $1,187.65  
   D. $535.00

10. What would Trevor's Lawn Service have to charge to cut a lawn if it took 107 lawns for them to break even?
    A. $27.85  
    B. $33.95  
    C. $38.95  
    D. $41.25
Assessment Solutions: Revenue, Expenses and the Break-Even Point

1. Define revenue.
   A. The price charged for a service
   B. Income in excess of expenses
   C. **Total income produced by selling a product or service**
   D. The income necessary to pay all expenses

2. What is the break-even point?
   A. **The point where revenue exactly equals expenses; there is zero profit**
   B. Expenses that are not affected by how many products are sold
   C. Expenses of producing or delivering each product or service
   D. The point where zero profit has been earned

3. The expenses of running a business are:
   A. Fixed and recurring
   B. Variable and constantly changing
   C. Unpredictable
   D. **A combination of fixed and variable per-unit**

4. Contribution margin is:
   A. Revenue available to only pay fixed costs
   B. Equal to fixed costs at the break-even point
   C. Calculated by subtracting variable costs from total revenue
   D. **All of the above**
5. With equipment and facility rental costs of $525/month and employee wages of $120/week per employee, Trevor’s Lawn Service has a contribution margin of $2,640 on monthly revenue of $3,175. What are Trevor’s Lawn Service’s monthly variable costs?

Revenue - Contribution margin = 3175 - 2640 = 535

A. $1,005  
B. **$535**  
C. Cannot be determined without knowing how many employees  
D. Cannot be determined without knowing the number of monthly sales (lawns mowed)

6. If Trevor’s Lawn Service employs four full-time workers, and they service 107 lawns each month, what are the per-lawn variable costs?

\[ \frac{535}{107} = 5 \]

A. **$5**  
B. $535  
C. $1,920  
D. $2,445

7. If Trevor’s Lawn Service charges $38.95 to cut a lawn, how many lawns represent the company's break-even point?

\[ FC = 525 \text{ (rental costs)} + (16 \times 120; \text{salary for 4 employees for 4 weeks}) = 2445 \]

\[ P = 38.95 \]

\[ V = 5 \]

\[ X = \frac{FC}{P - V} = \frac{2445}{33.95} = 72.02 \]

A. 63  
B. 19  
C. **72**  
D. 30
8. How much revenue would it take for Trevor’s Lawn Service to break even if they cut 107 lawns?

\[ 2445 + (107 \times 5) = 2980 \]

A. $3,632.65  
B. $2,804.40  
C. $2,444.40  
D. $2,980.00  

9. What would Trevor’s Lawn Service’s profit be if they charged $38.95/lawn and cut 107 lawns?

Revenue = 107 x 38.95 = 4167.65  
Profit = Revenue – Expenses = 4167.65 – 2980 = 1187.65  
A. $1,723.25  
B. $1,363.25  
C. $1,187.65  
D. $535.00  

10. What would Trevor’s Lawn Service have to charge to cut a lawn if it took 107 lawns for them to break even?

\[ P(107) = 2980 \quad P = \frac{2980}{107} = 27.85 \]

A. $27.85  
B. $33.95  
C. $38.95  
D. $41.25